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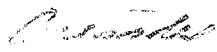
## LIQUID OSCILLATION FREQUENCIES IN TILTED CYLINDERS FOR FIVE CROSS-SECTION SHAPES

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# DEFINITION OF SYMBOLS

SYMBOL	DEFINITION
d	Distance from tank axis to flat tank wall, cm (see figure 2)
g	Standard acceleration, 980.665 cm/sec <sup>2</sup>
h	Liquid depth, measured along the tank axis, cm
R	Radius of circular part of cross section, cm
$\alpha$	Tilt angle, deg (see figure 2)
$\omega$	Liquid oscillation frequency, rad/sec
$\frac{\omega^2 R}{g}$	Nondimensional frequency parameter

# LIQUID OSCILLATION FREQUENCIES IN TILTED CYLINDERS FOR FIVE CROSS-SECTION SHAPES

## SUMMARY

The frequency of liquid oscillations was measured in cylindrical tanks with cross-section shapes formed by a circular arc and a straight line. The shapes were related to some feasible and proposed Space Shuttle tank cross sections and ranged from circular to approximately semi-circular. The frequencies were measured with the tank axes tilted  $0^\circ$  to  $60^\circ$  from the local vertical. Oscillations were excited by both forces in the plane of the tilt angle and forces perpendicular to the plane of the tilt angle.

The data were compared with theoretical results for frequency in tilted circular cylindrical tanks and theoretical results for one upright cylindrical tank with non-circular cross section.

## I. INTRODUCTION

Knowledge of the natural oscillation frequencies of liquid propellant and the forces exerted by the liquid on its tank wall is required for successful design of control systems for liquid-fueled rocket vehicles, since at launch 80 percent or more of the total vehicle mass is liquid. The propellant dynamics at high Bond number for liquid in axisymmetric tanks has been studied at length and is well understood.

Interest in propellant dynamics in tanks of various cross-section shapes with their axes inclined from the total acceleration vector has been generated by studies of the Space Shuttle orbiter and booster. Candidate reusable Space Shuttle configurations are two-stage aircraft-type vehicles. The asymmetry of the launch vehicle causes an angle of  $0^\circ$  to  $10^\circ$  between the thrust vector and the tank axes during launch, and during the flyback phase of flight, larger tank tilt angles will exist. Most of the propellant tanks are cylindrical or conical and have circular cross sections. A cross section formed by two equal-size circular arcs having a common chord (referred to as double lobe) has been proposed in some orbiter configurations.

References 1 through 4 present analyses of liquid oscillation in tilted circular cylinders. References 1 through 3 give frequency results, and reference 4 gives force, moment, and damping results in

addition to frequency. Reference 5 includes a theoretical analysis of frequencies and mode shapes in an upright double lobe tank. The purpose of the present study is to determine experimentally the frequency of liquid oscillations in tanks tilted  $0^\circ$  to  $60^\circ$  from the vertical and having a series of cross-section shapes formed from a circle and a chord. Each of these shapes represents a symmetric half of the double lobe shape. The experimental results of this study are compared with theory from references 1, 2, 4 and 5.

## II. APPARATUS AND PROCEDURE

The basic test structure is shown in figure 1. The base of the structure is aluminum, and the cylinder is plexiglass. The inside radius of the cylinder is 10.17 cm, and the height is 38.1 cm. The test liquid was distilled water and aerosol MA (a wetting agent) in a .2 percent concentration. Noncircular cross sections were formed by clamping one of four aluminum plates inside the cylinder parallel to the axis. The designation of the cross-section shapes by the dimensionless ratio,  $d/R$ , and by shape number is shown in figure 2. Equipment not shown consisted of a stopwatch, a c-clamp, and triangles with  $5^\circ$ ,  $8^\circ$ ,  $10^\circ$ ,  $30^\circ$ ,  $45^\circ$ , and  $60^\circ$  angles.

On the first few runs, a plate (Shape 4) was positioned as in orientation 2 of figure 2 at  $\theta = 10^\circ$ . This was abandoned for orientation 1 throughout the rest of the test. The cylinder was filled to one of the certain depths to be tested. The frequency was measured for each depth with the tank tilted  $0^\circ$ ,  $10^\circ$ ,  $45^\circ$ , and  $60^\circ$ . For the circular shape,  $5^\circ$  and  $8^\circ$  tilts were also used. The tilt angles were set by placing the appropriate triangle between the cylinder base and the horizontal base of the support structure, then tightening the screws to lock the frame.

The test fluid was set into motion in either the lateral or longitudinal direction (defined in figure 2) by raising and rocking one end of the base in the preferred direction. The fluid was agitated slowly until it reached the first mode of oscillation and crested approximately one-half inch above the normal surface plane. At this time, twenty-five cycles of oscillation were timed by the stopwatch while the oscillation decayed.

## III. DISCUSSION

The excitation directions for the longitudinal and lateral modes of liquid oscillation are shown in figure 2. The longitudinal oscillation is excited by a force in the plane of the tilt angle and the

lateral by a force perpendicular to the plane of the tilt angle. In each case, the purpose was to excite the first antisymmetric slosh mode.

The double lobe tank arrangement in the Shuttle orbiter is such that the angle  $\alpha$  in orientation 1 of figure 2 represents a tilt of the tank in the pitch plane, and  $\alpha$  in orientation 2 is tilt in the yaw plane. The majority of data were taken for orientation 1.

#### A. Longitudinal Oscillations

The data for the longitudinal oscillations, orientation 1, are presented in figures 3a - 3e. The frequency parameter is shown as a function of liquid depth and tilt angle for the five cross-section shapes. The data for the circular cross section (figure 3a) were taken from reference 3 except for the four values at  $h/R > 3$ . The frequency at  $h/R = 3.25$ ,  $\alpha = 0^\circ$ , which was used as a check point, was found to be very near the theoretical value and to agree well with the experimental data of reference 3. There was only a slight reduction of frequency for tilt angles of  $10^\circ$  or less (the range of tilt likely to exist during Shuttle boost flight).

The frequencies for Shape 1,  $d/R = .678$ , are shown in figure 3b. These frequencies are slightly lower than those for the circular cross section, but the curves are otherwise very similar, approaching a constant value as depth increases. Shape 1 is closest of the shapes tested to double lobe shapes on proposed orbiter configurations. Khabbaz, in reference 5, determined theoretical frequencies for several modes of liquid oscillation in a double lobe tank with  $d/R = .71$ . A theoretical point from his work is shown as the solid symbol in figure 3b for  $\alpha = 0^\circ$ .

The frequency curves for Shapes 2, 3, and 4, shown in figures 3c, d, and e, respectively, varied with liquid depth and tank tilt similarly to those for the circular cross section except that at some angles the frequency showed a slight decrease with increasing depth.

Figure 4 shows the longitudinal frequency versus tilt angle for all tank shapes tested. The data are for  $h/R \geq 1.75$  so that the frequencies are nearly independent of increases in depth. The magnitude of frequency change with tank shape is seen to be greater at  $0^\circ$  than at  $60^\circ$ .

The frequency parameter is plotted against a cross-section shape parameter,  $d/R$  in figure 5.  $d/R = 0$  corresponds to a semi-circular cross section and  $d/R = 1.0$  represents the circular cross section. For the longitudinal modes, the frequencies for  $d/R = 0$  and  $d/R = 1.0$



should be the same; therefore, the values for  $d/R = 1.0$  were replotted as the solid symbols at  $d/R = 0$ . The maximum variation of frequency for this range of shapes was 4.5 percent at  $\alpha = 45^\circ$ . At  $\alpha = 0^\circ$ , the frequency decreased about 2.6 percent due to cross-section change. The frequencies at  $60^\circ$  tilt were less than those at  $0^\circ$  by about 48 percent throughout the shape range.

## B. Lateral Oscillations

Excitation of the lowest lateral mode was difficult. The mode shape changed in some instances with changes in tilt angle, liquid depth, or during decay of an oscillation. The lateral modes had generally higher frequencies than the longitudinal modes. The instability of modes and the higher frequencies decreased the accuracy of the frequency measurements. Several measurements were made at each test condition, and the average deviation from the mean was .5 percent for the longitudinal modes and 2.1 percent for the lateral modes.

Figure 6 shows the variation of lateral mode frequency with liquid depth. The data for figure 6a were taken from reference 3 except for the values at  $\alpha = 5^\circ$ ,  $8^\circ$ , and  $10^\circ$ . The change in tilt angle is seen to have much less effect for the lateral modes than for the longitudinal. For the shapes other than circular, figures 6b - 6e, the frequency did not consistently decrease with increasing tilt angle or decreasing depth. Two distinct frequencies were found at  $h/R = 1.75$  in figure 6c for  $\alpha = 45^\circ$ . Also at  $h/R = 1.75$ , the frequency for  $\alpha = 60^\circ$  was higher than that for one of the frequencies for  $\alpha = 45^\circ$ . Only one frequency was found at each test condition for Shapes 3 and 4, figures 6d and 6e, respectively, but it is not clear whether the frequencies represent the first or some higher lateral mode.

Figure 7 is a cross plot of data from figure 6 showing the variation of frequency with cross-section shape and tilt angle at one liquid level. The change in cross section is seen to have a larger effect on frequency than change in tilt angle. The opposite was true for the longitudinal slosh (figure 5).

The lateral oscillation frequencies at  $\alpha = 0^\circ$  are shown for the various cross sections in figure 8 as a function of liquid depth. A theoretical value of frequency parameter from reference 5 for a shape with  $d/R = .71$  is compared with experiment for Shape 1,  $d/R = .678$ , and seen to be slightly higher. Figure 9 shows the data taken for orientation 2.

### C. Comparison With Theory

A summary of theoretical results for longitudinal frequency of liquid oscillation in a tilted cylinder with circular cross section is presented in figure 10 and compared with experimental data. The theoretical values are seen to be generally lower than experiment with good agreement at small tilt angles. The maximum deviation of theoretical frequency from experiment was about 13 percent.

### CONCLUSIONS

Frequencies of liquid oscillations in cylindrical tanks were measured for a range of cross-section shapes and tilt angles, for two excitation directions, and two tilt directions. The majority of data were for one tilt direction, a tilt such that the flat wall of the tank lay in the plane of the tilt angles. The conclusions presented below are for this tilt direction.

With the excitation force in the plane of the tilt angle, the frequency of liquid oscillation reached a minimum value about 4.5 percent lower than that for the circular cross-section tank as the cross-section shape was varied from circular to near semi-circular. The circular and semi-circular tanks had about the same liquid frequency. The tilt angle had more effect than cross-section shape for this excitation direction, with the frequency at 60° tilt as much as 48 percent less than at 0°.

Excitation perpendicular to the tilt angle plane gave frequencies which increased sharply with change in cross-section shape from circular to near semi-circular. For this excitation direction, the change in tank shape caused the larger effects on frequency, and the decrease in frequency with increasing tilt was relatively small. Abrupt changes in frequency with changes in liquid depth, tank shape, and tilt angle were observed for excitation force perpendicular to the tilt angle plane, indicating that higher modes than the first were being excited.

The data were compared with available theoretical results. The theoretical values deviated from experiment by 0 percent to 13 percent with the better agreement generally occurring at low tilt angles in the circular cross-section tank.

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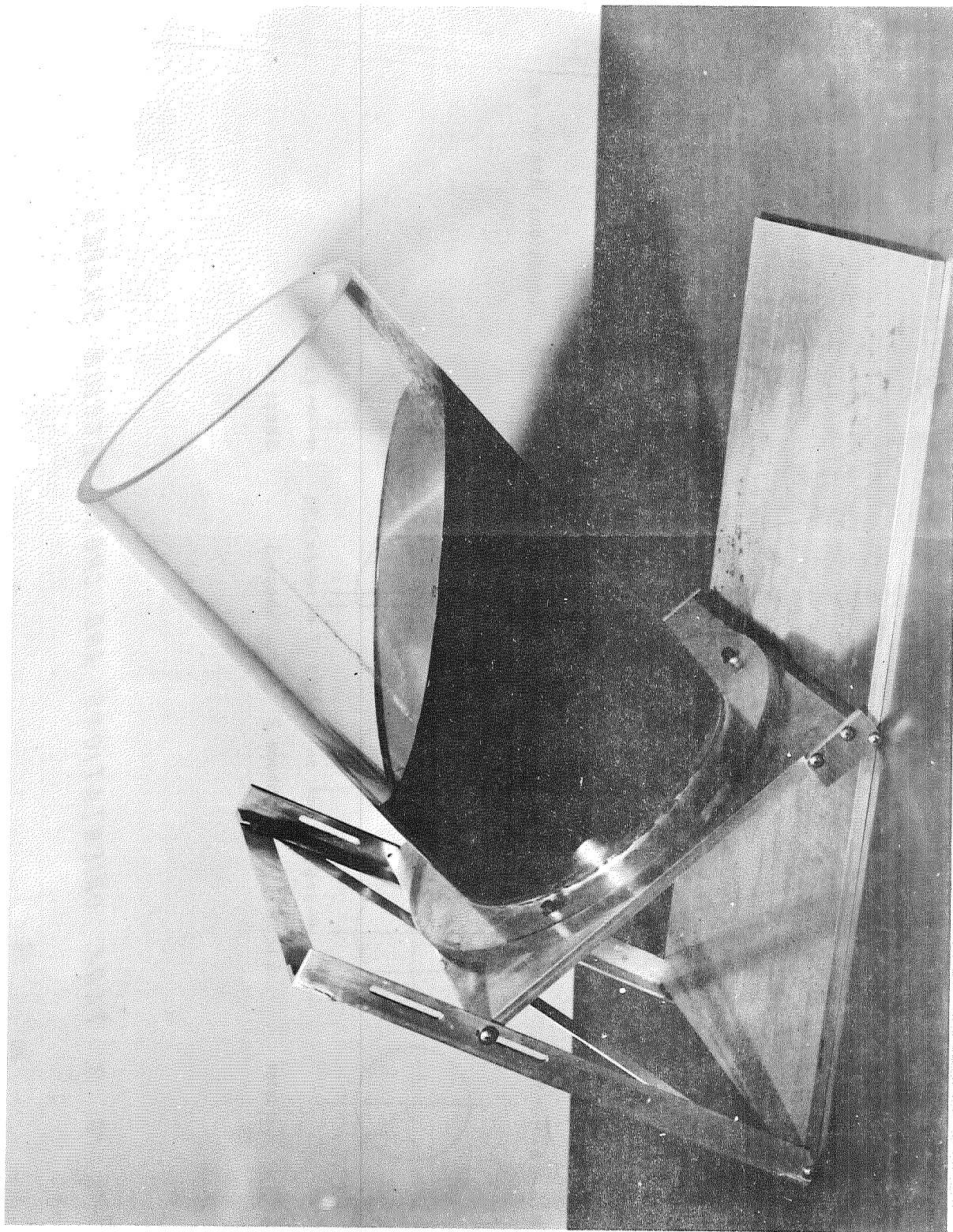


FIG. 1. TEST SET-UP

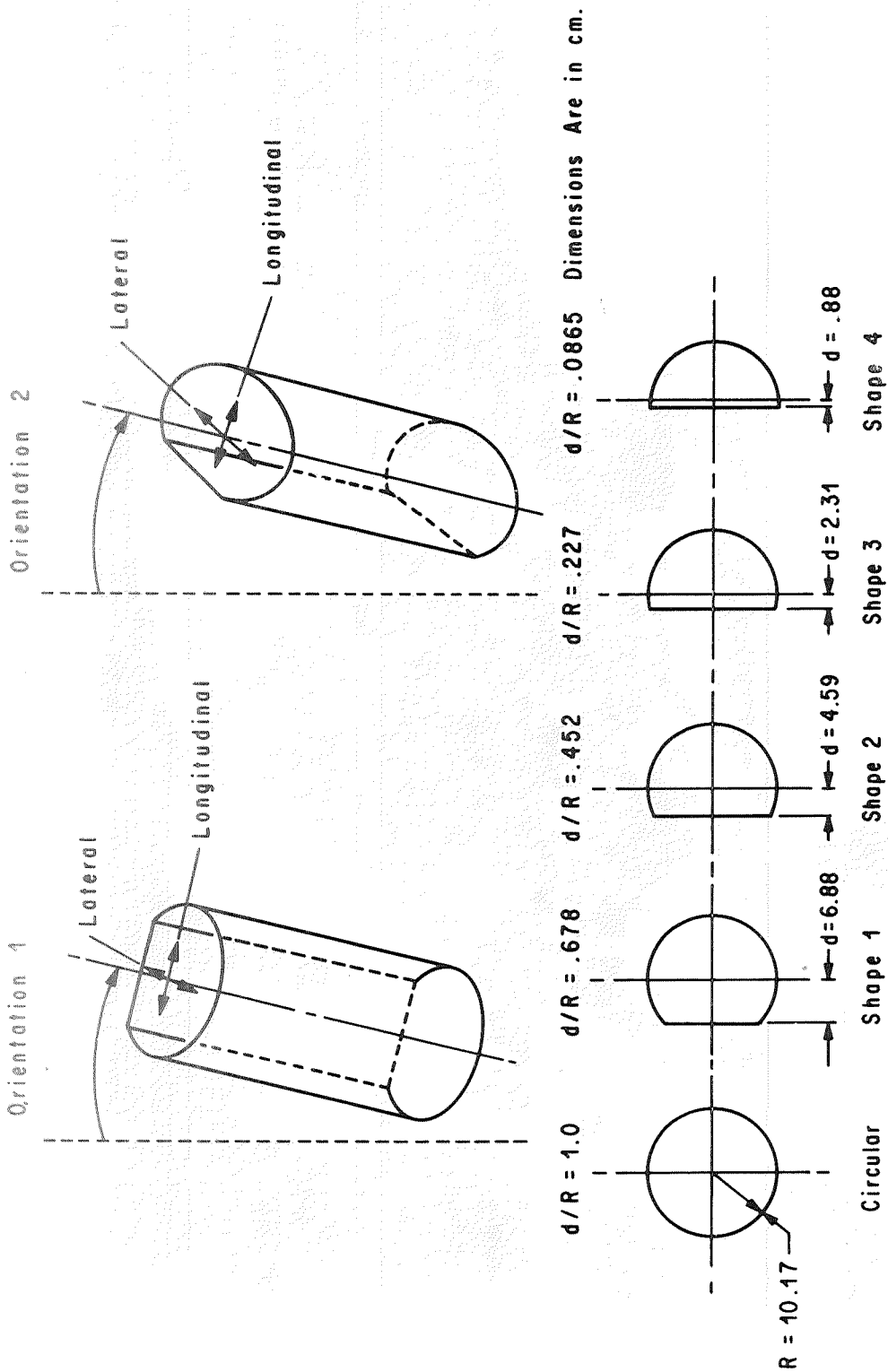


FIG. 2. TANK ORIENTATIONS AND CROSS SECTION SHAPES

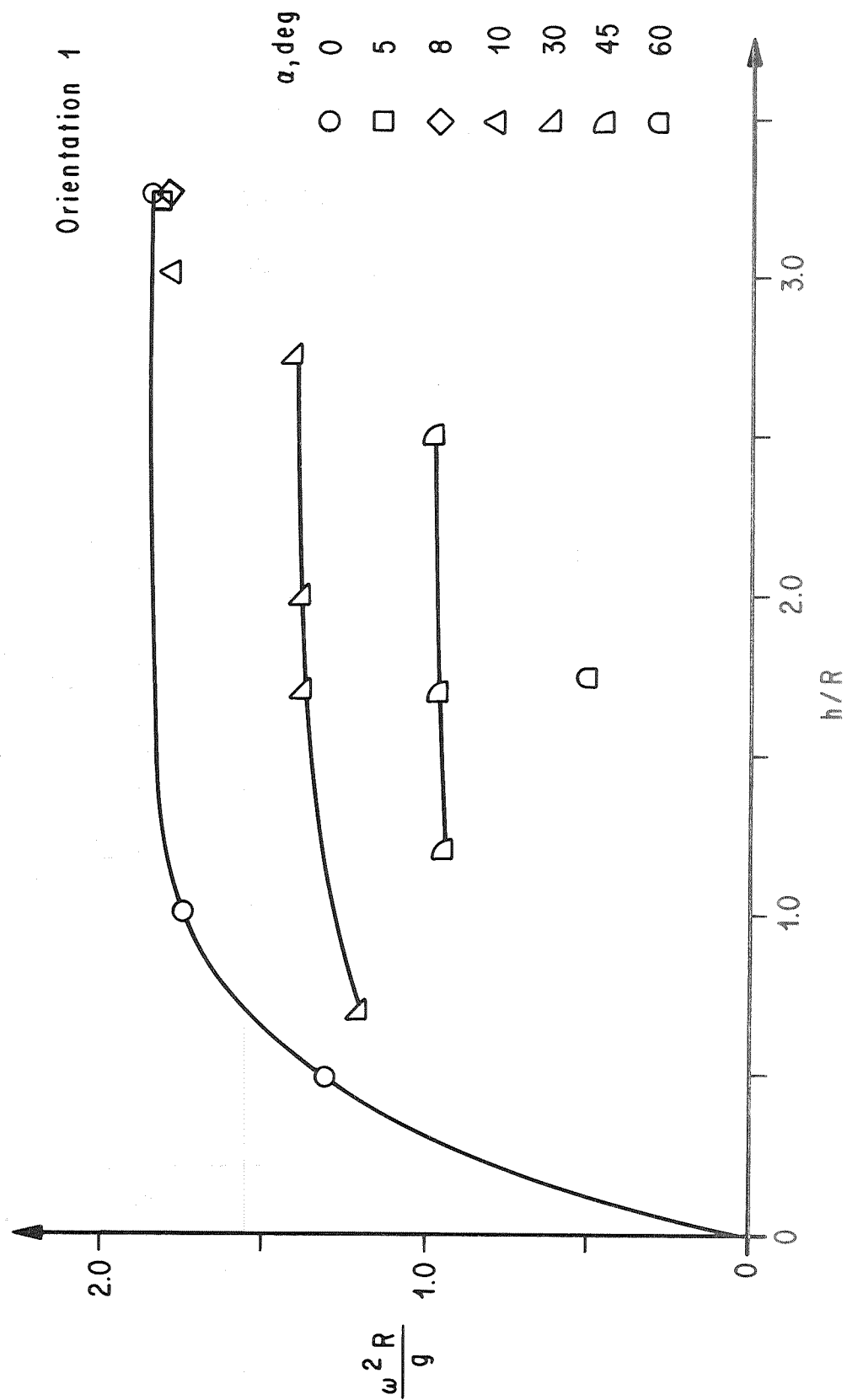


FIG. 3. EFFECT OF LIQUID DEPTH ON LONGITUDINAL FREQUENCY  
a. CIRCULAR

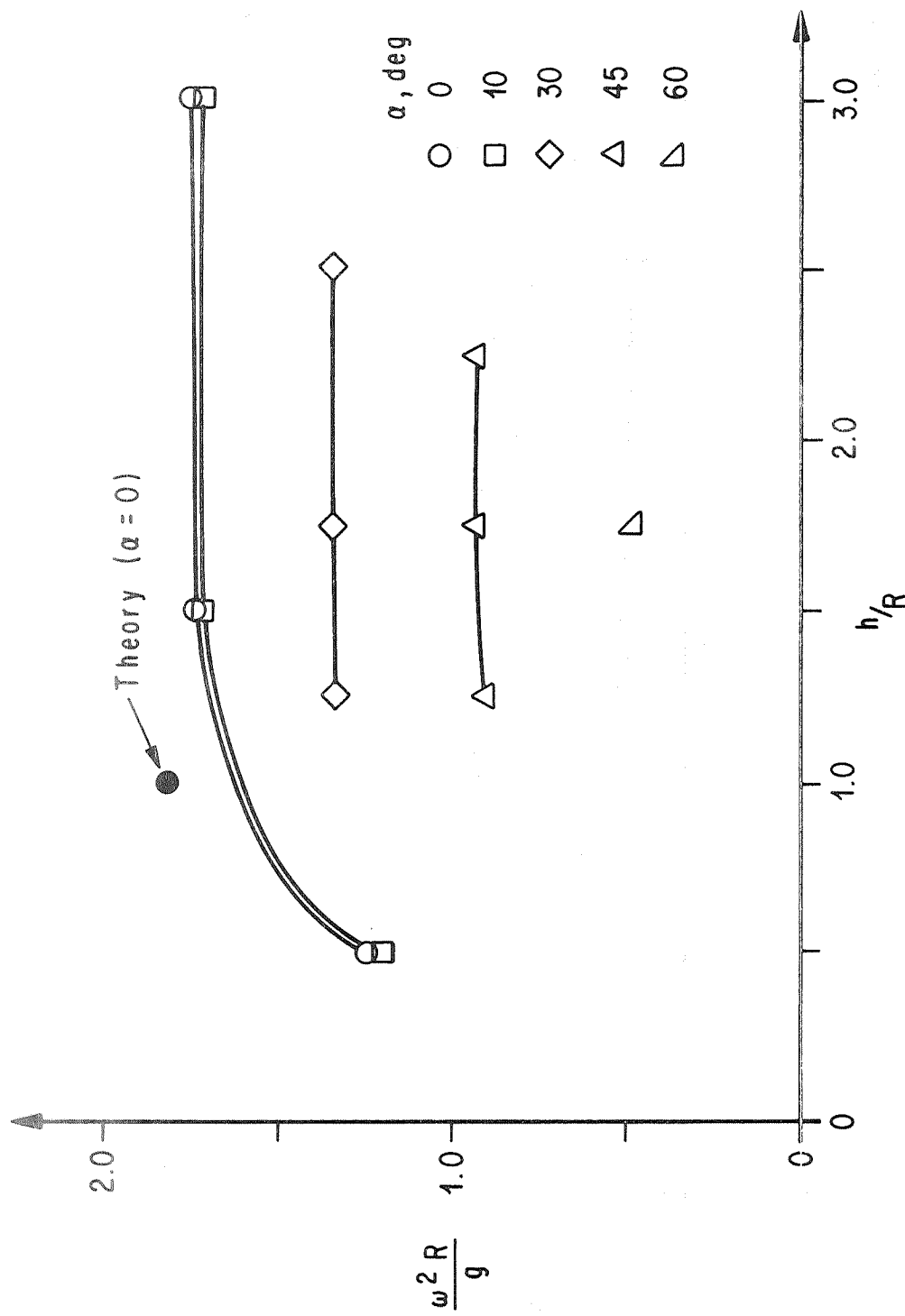


FIG. 3. - CONTINUED

b. SHAPE 1

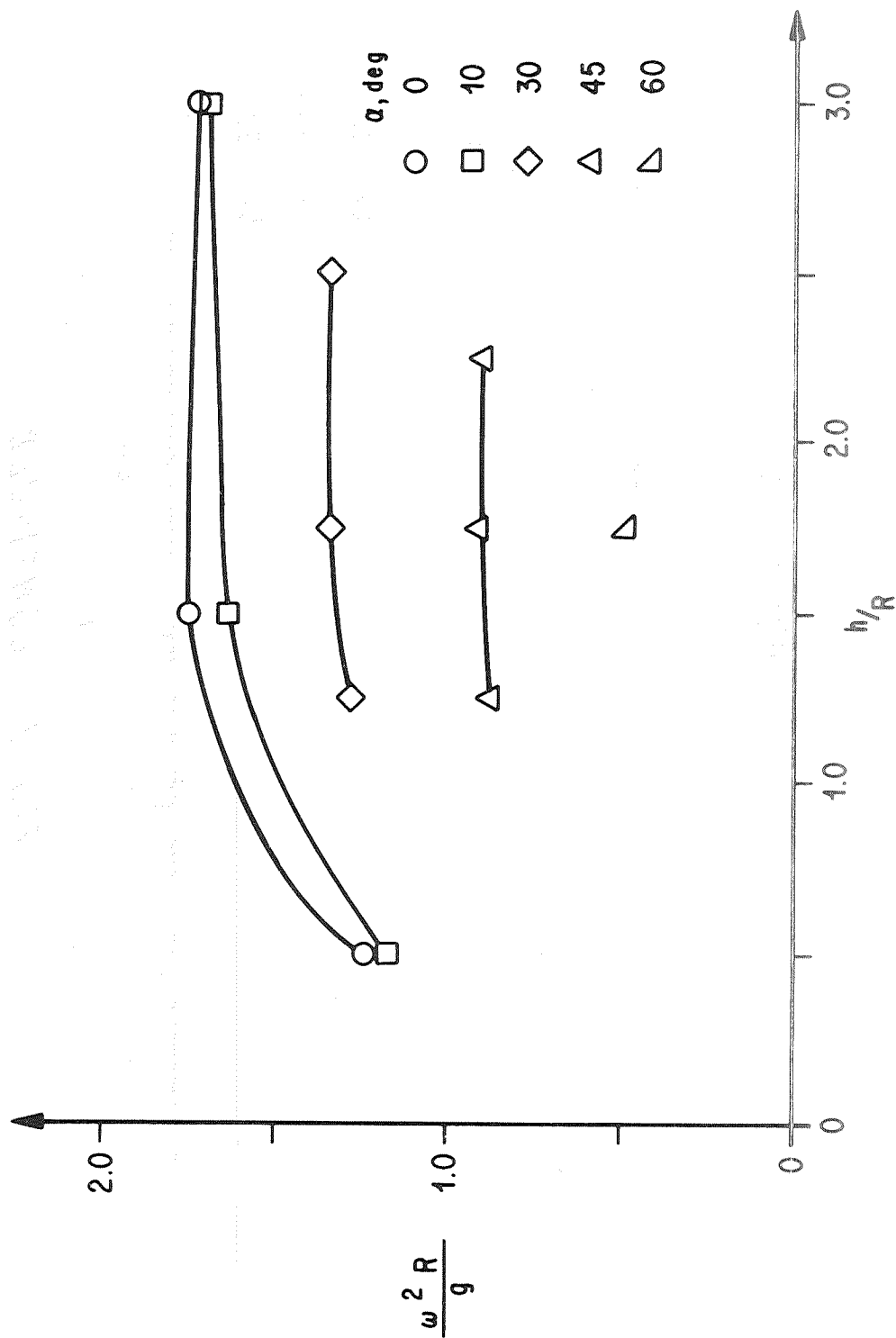


FIG. 3. - CONTINUED

c. SHAPE 2



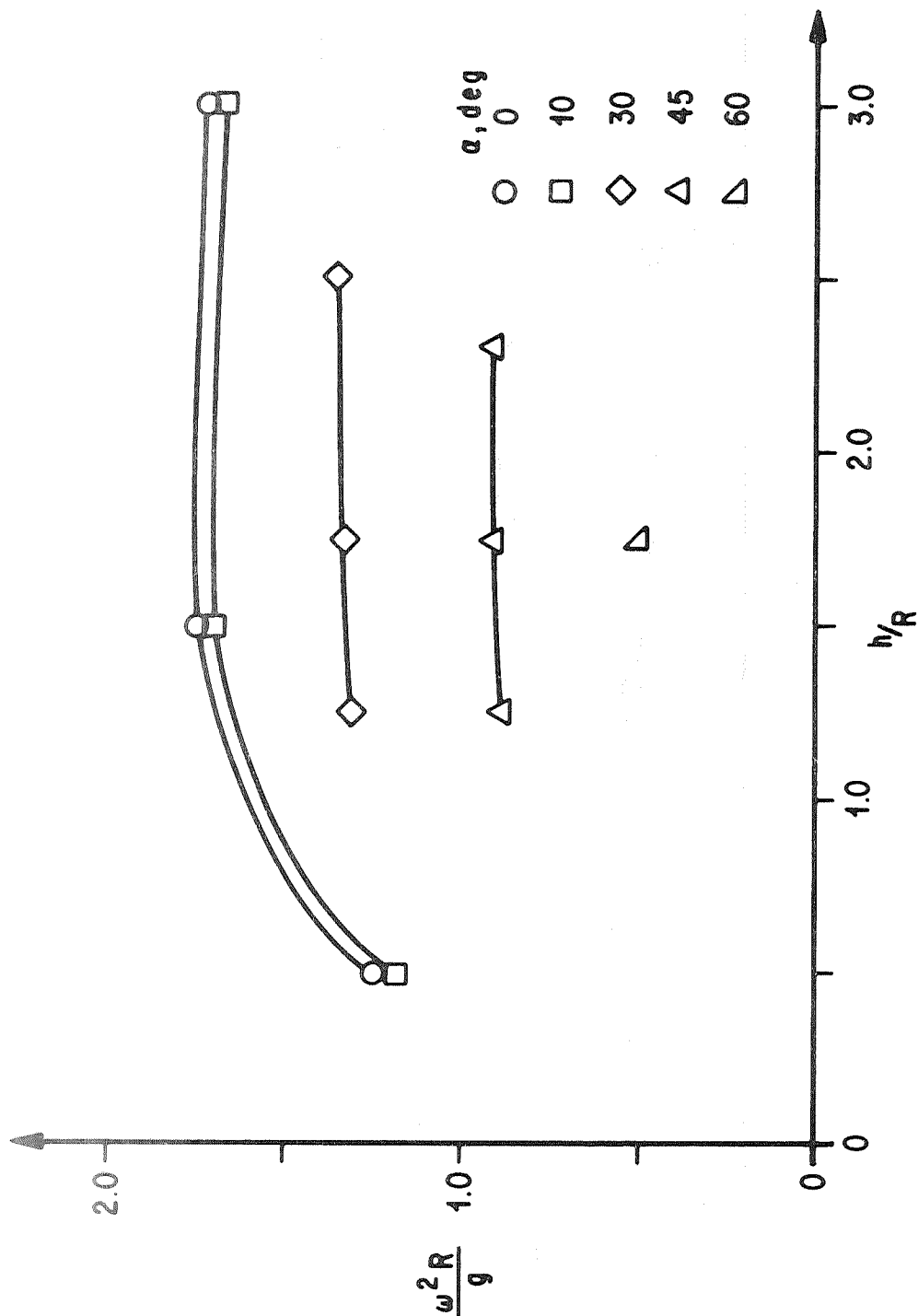


FIG. 3. - CONTINUED

d. SHAPE 3

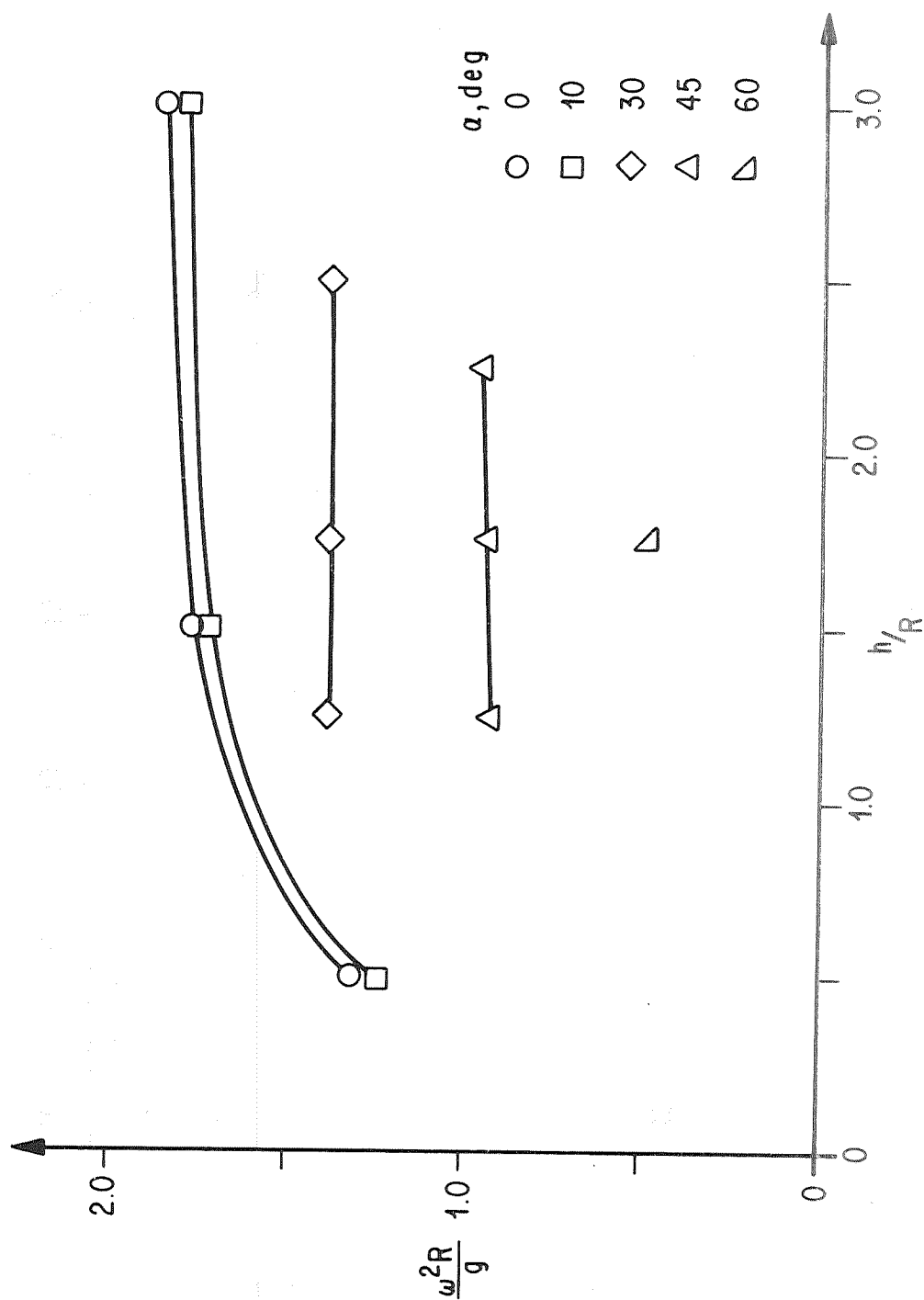


FIG. 3. - CONCLUDED

e. SHAPE 4

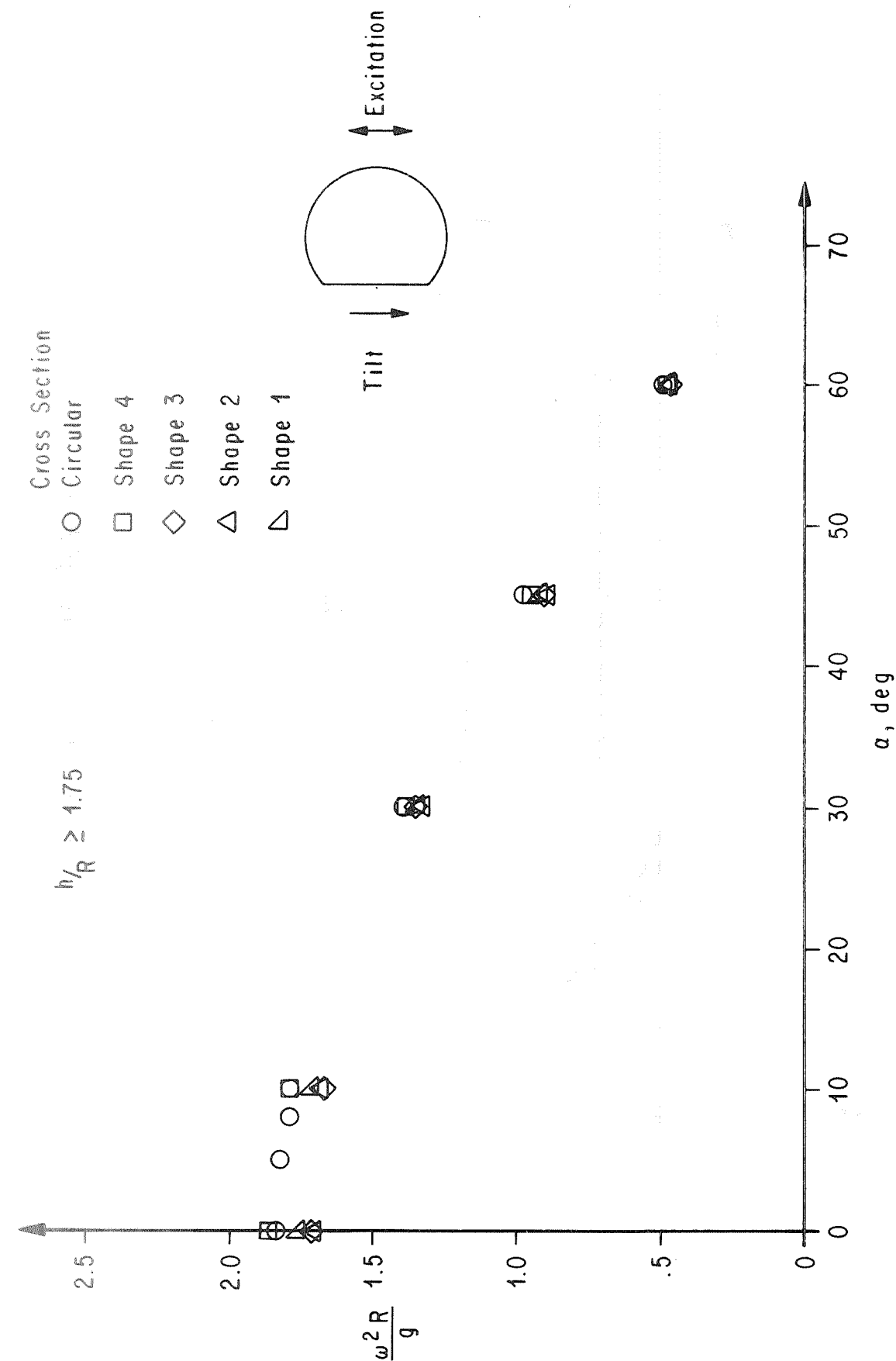


FIG. 4. - EFFECT OF TILT ANGLE ON LONGITUDINAL FREQUENCY

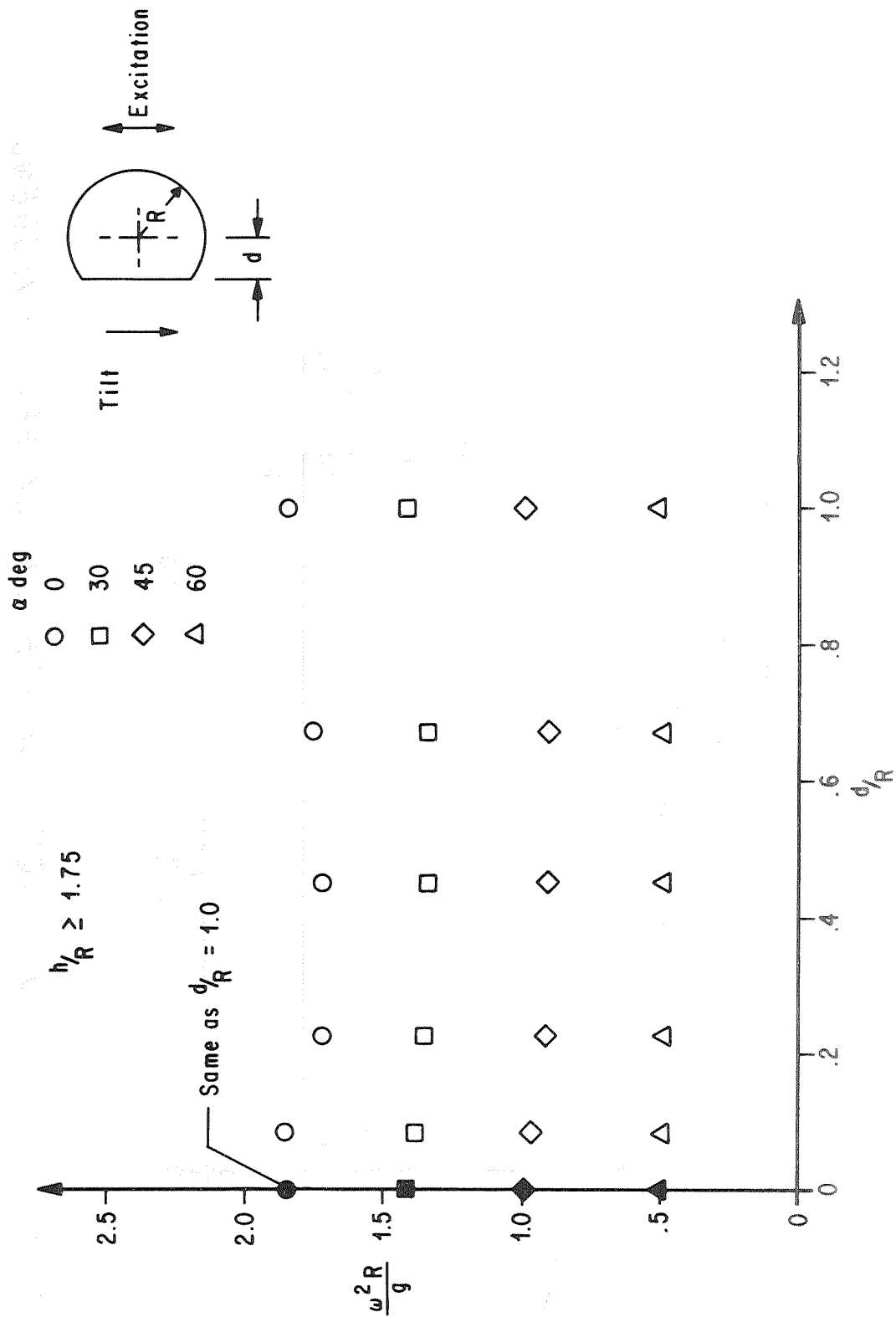


FIG. 5. - EFFECT OF CROSS SECTION SHAPE ON LONGITUDINAL FREQUENCY

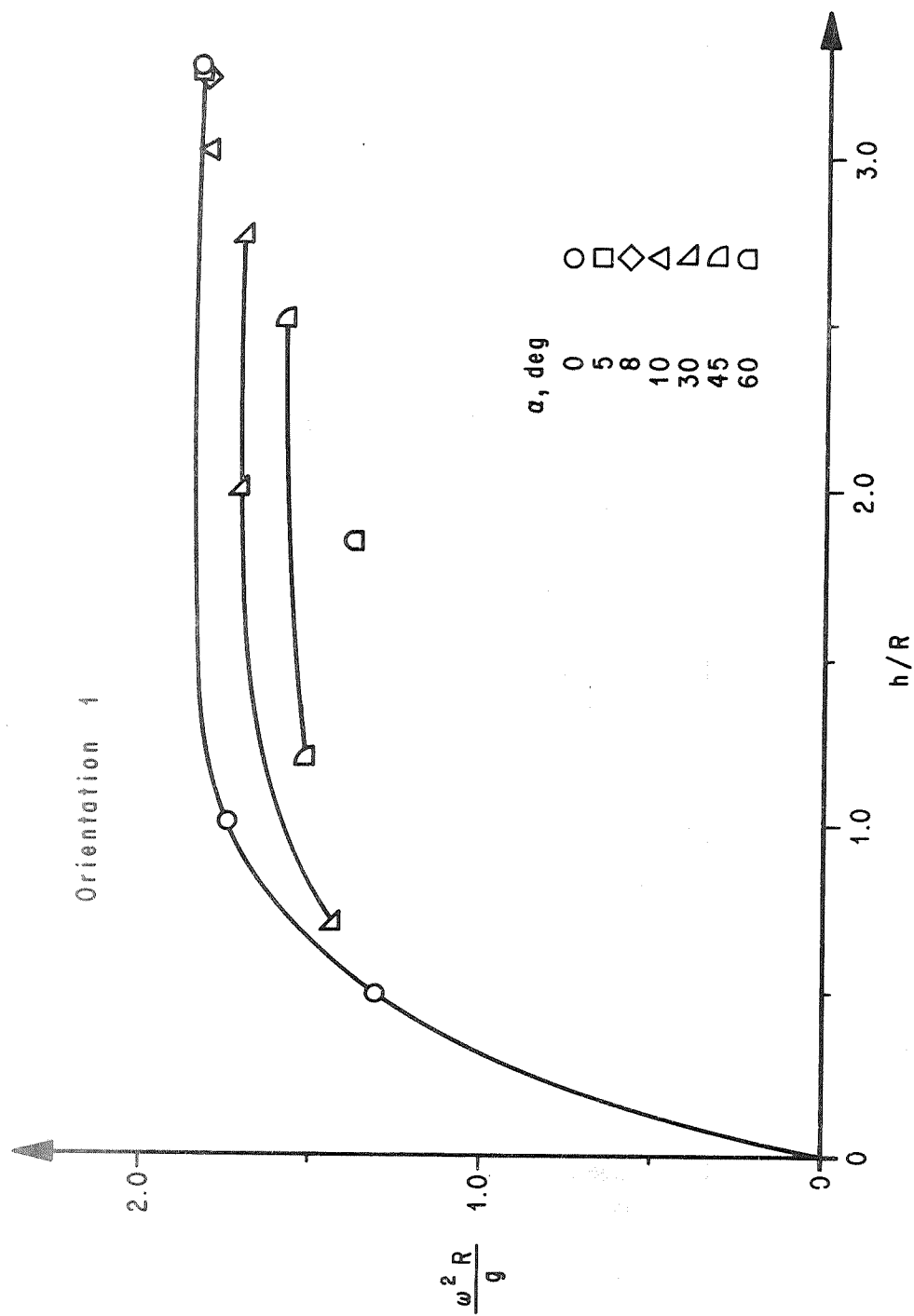


FIG. 6. EFFECT OF LIQUID DEPTH ON LATERAL FREQUENCY  
a. CIRCULAR

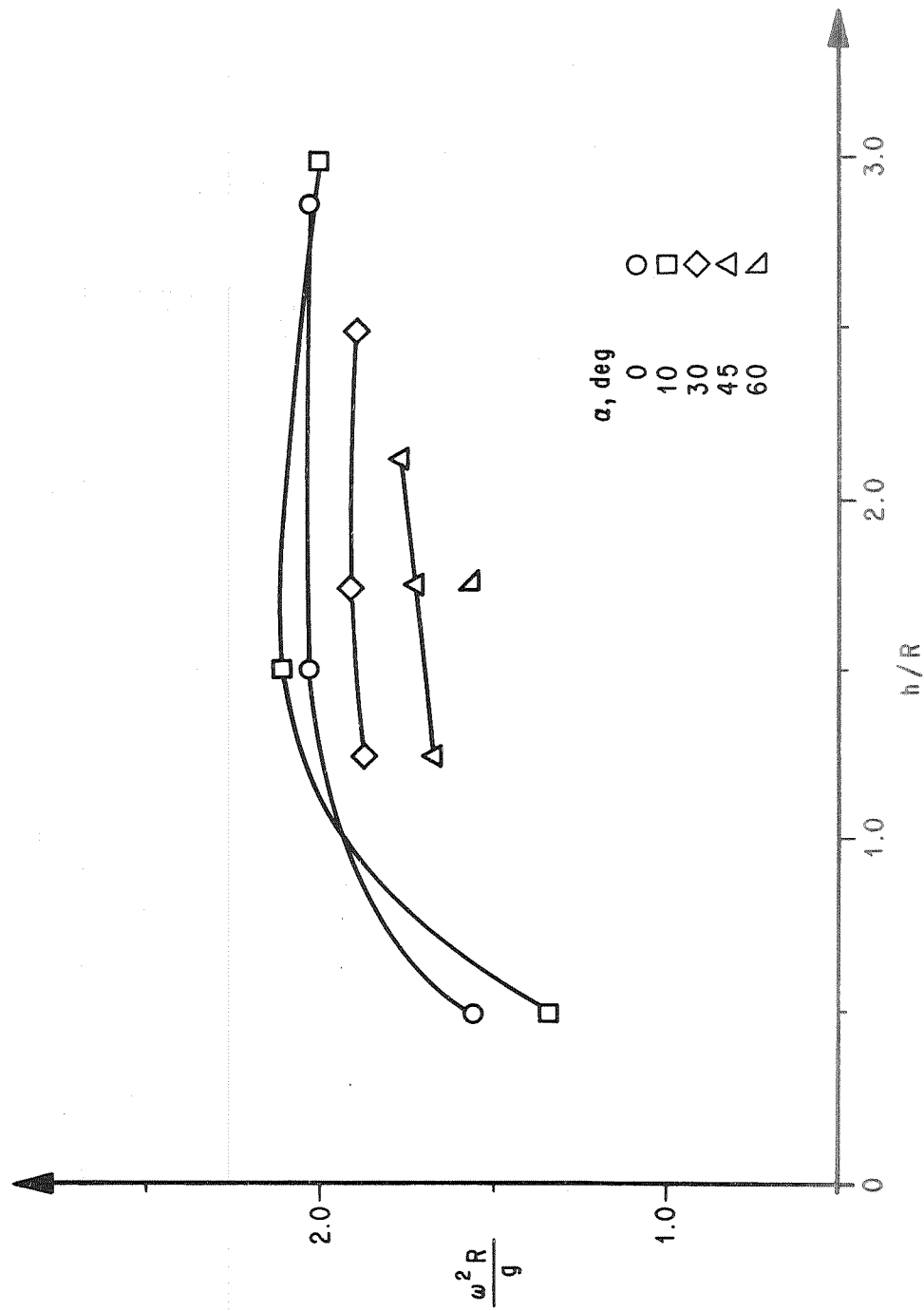


FIG. 6. Continued  
b. SHAPE 1

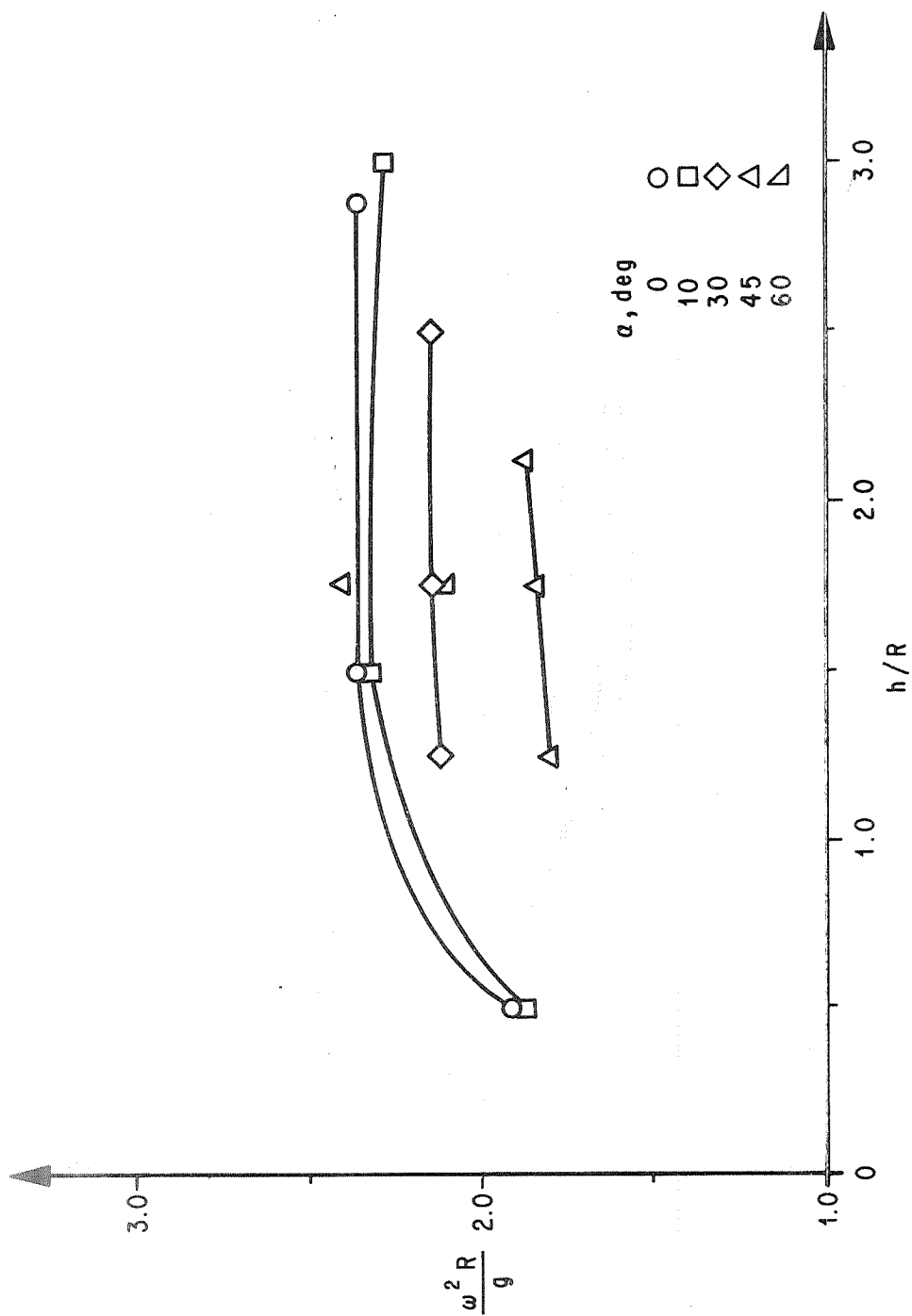


FIG. 6. Continued  
c. SHAPE 2

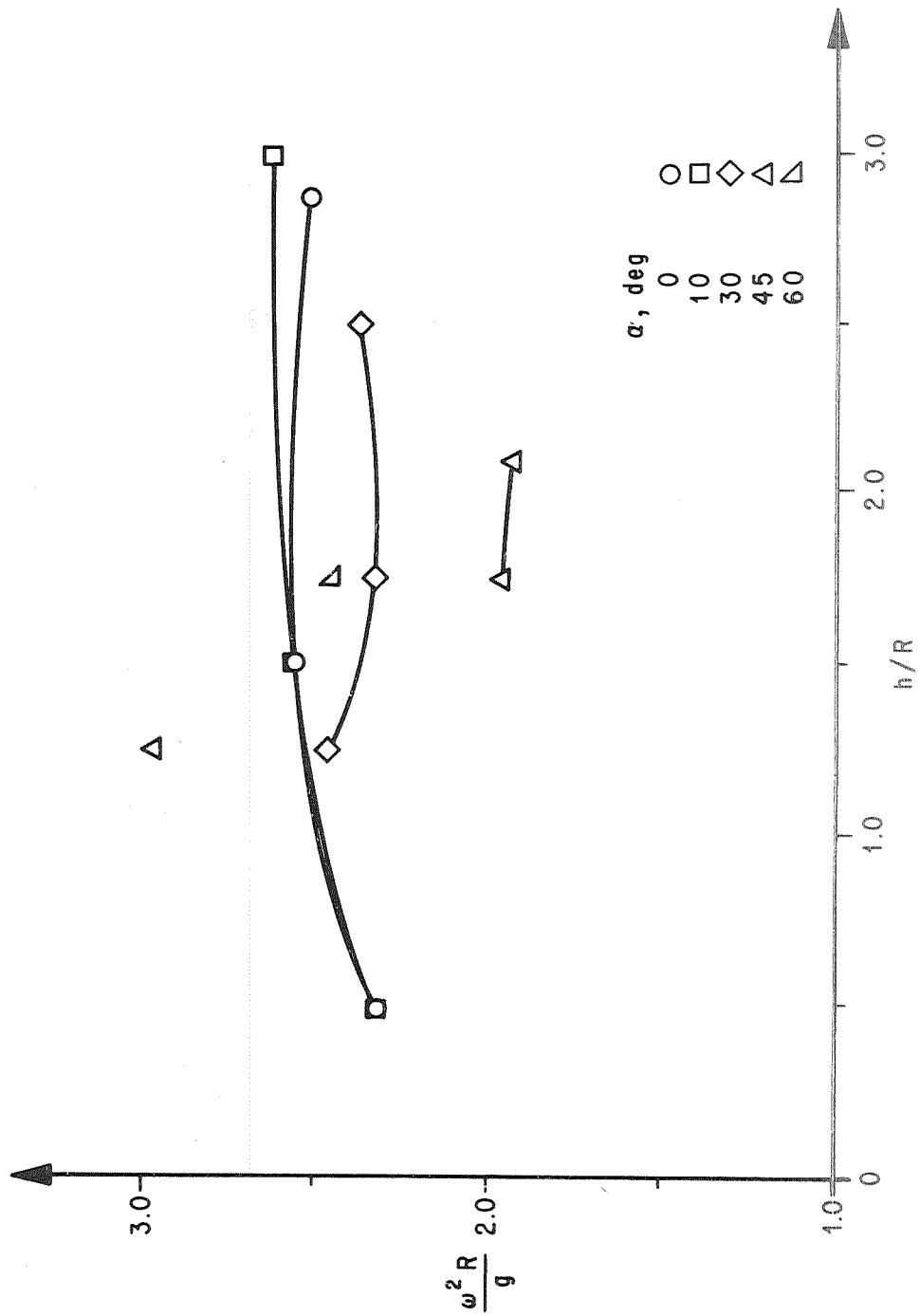


FIG. 6. Continued  
d. SHAPE 3



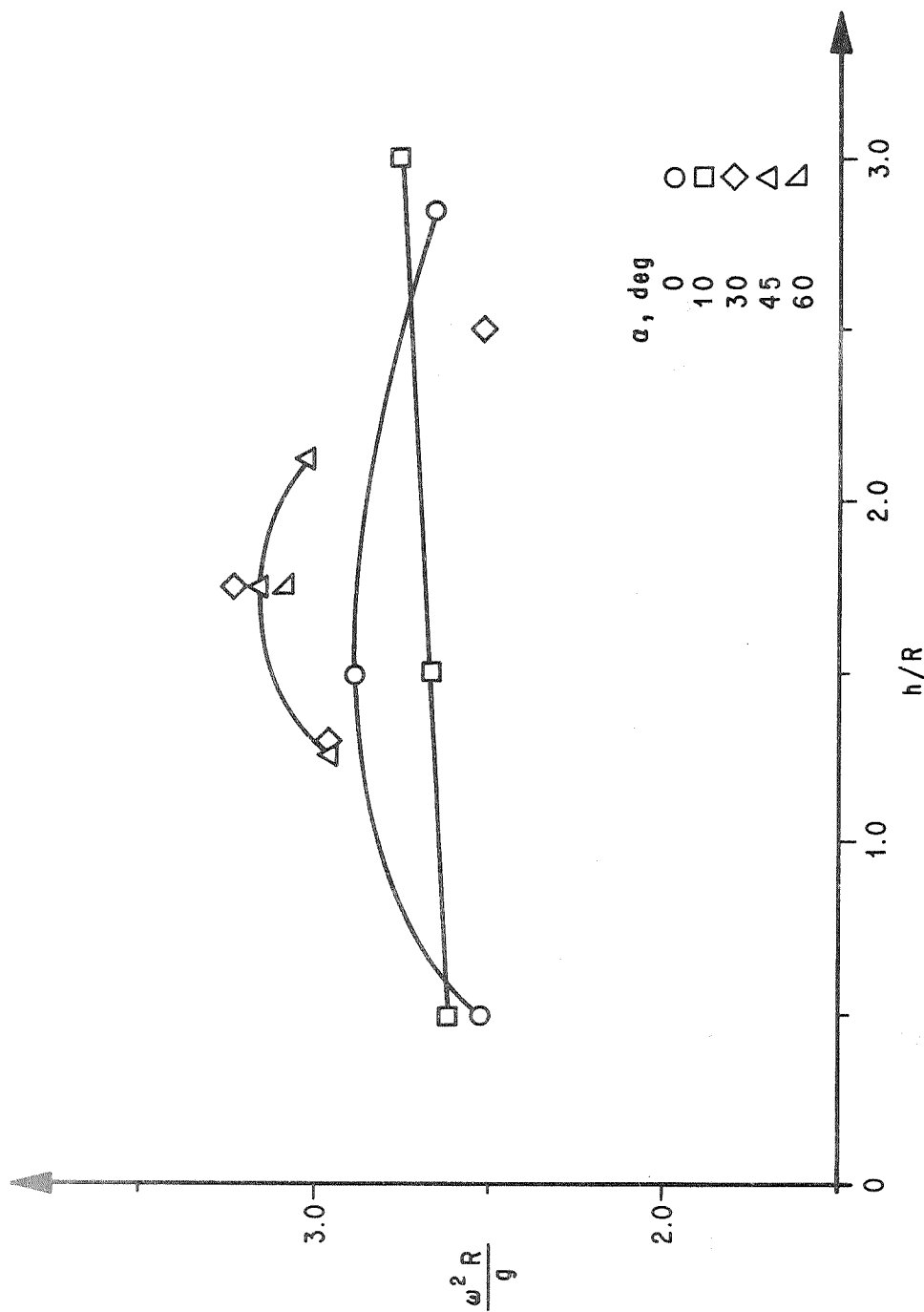


FIG. 6. Concluded  
e. SHAPE 4

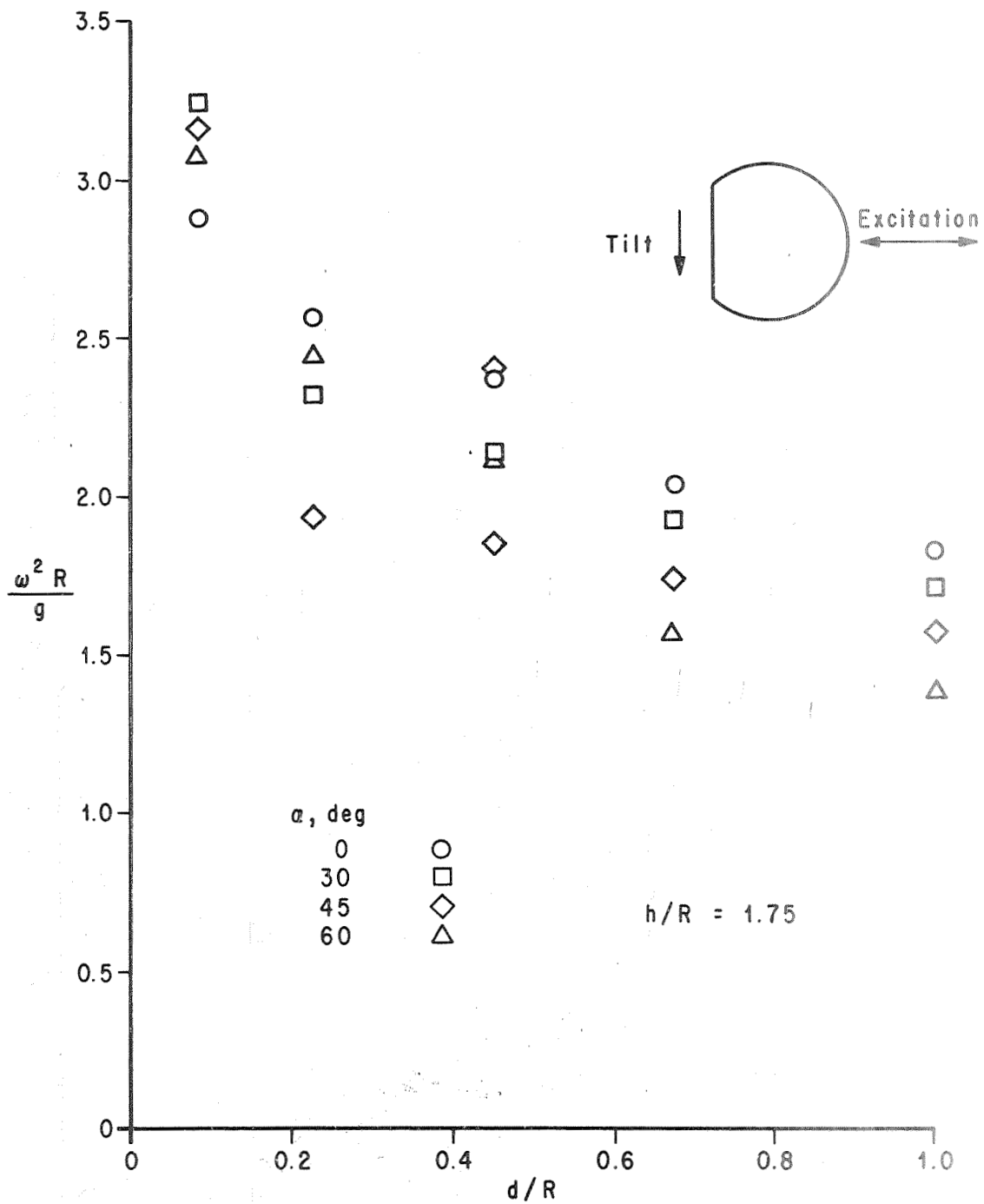


FIG. 7. EFFECT OF CROSS SECTION SHAPE ON LATERAL FREQUENCY

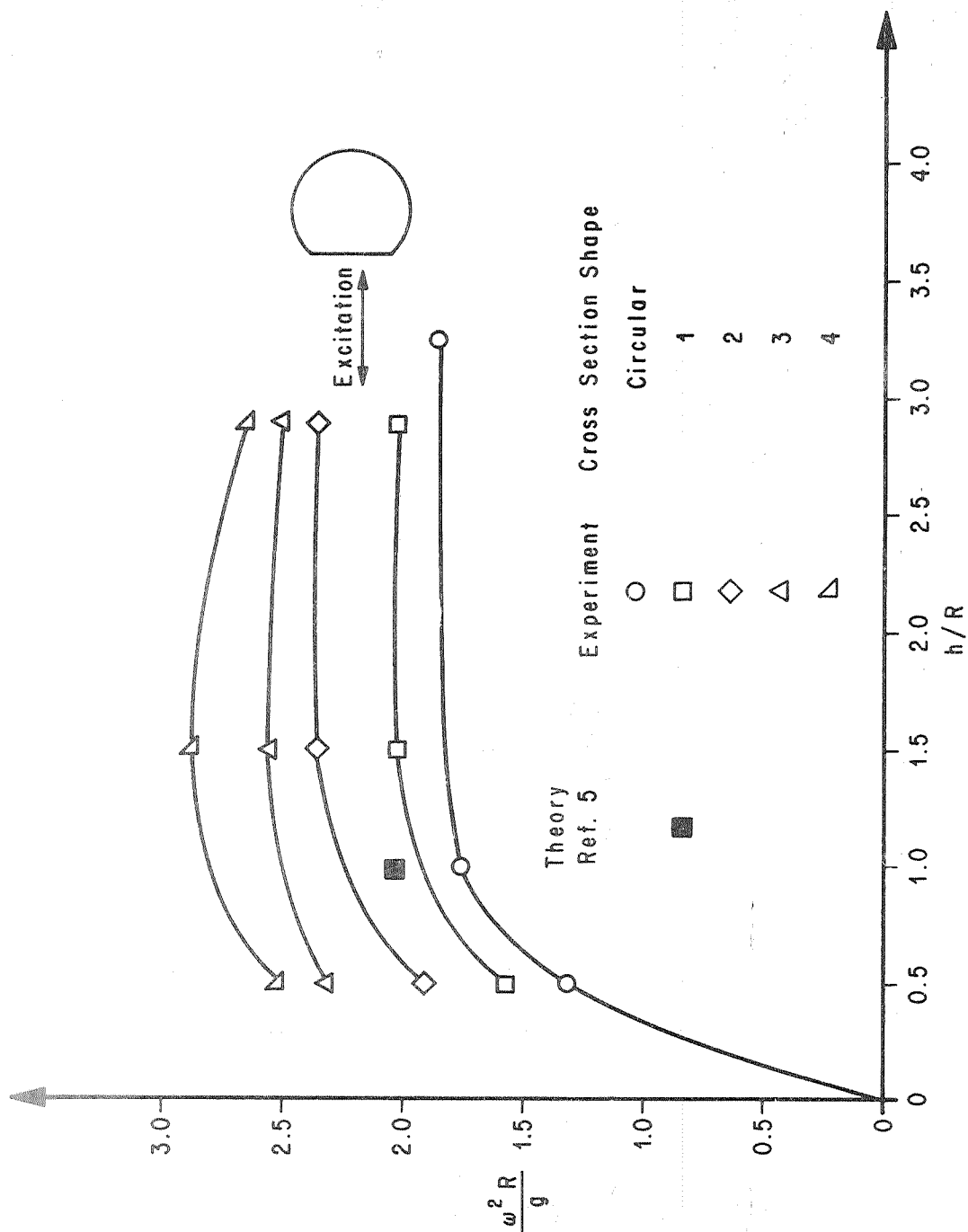


FIG. 8. EFFECT OF LIQUID DEPTH ON LATERAL FREQUENCY FOR  $\alpha = 0^\circ$

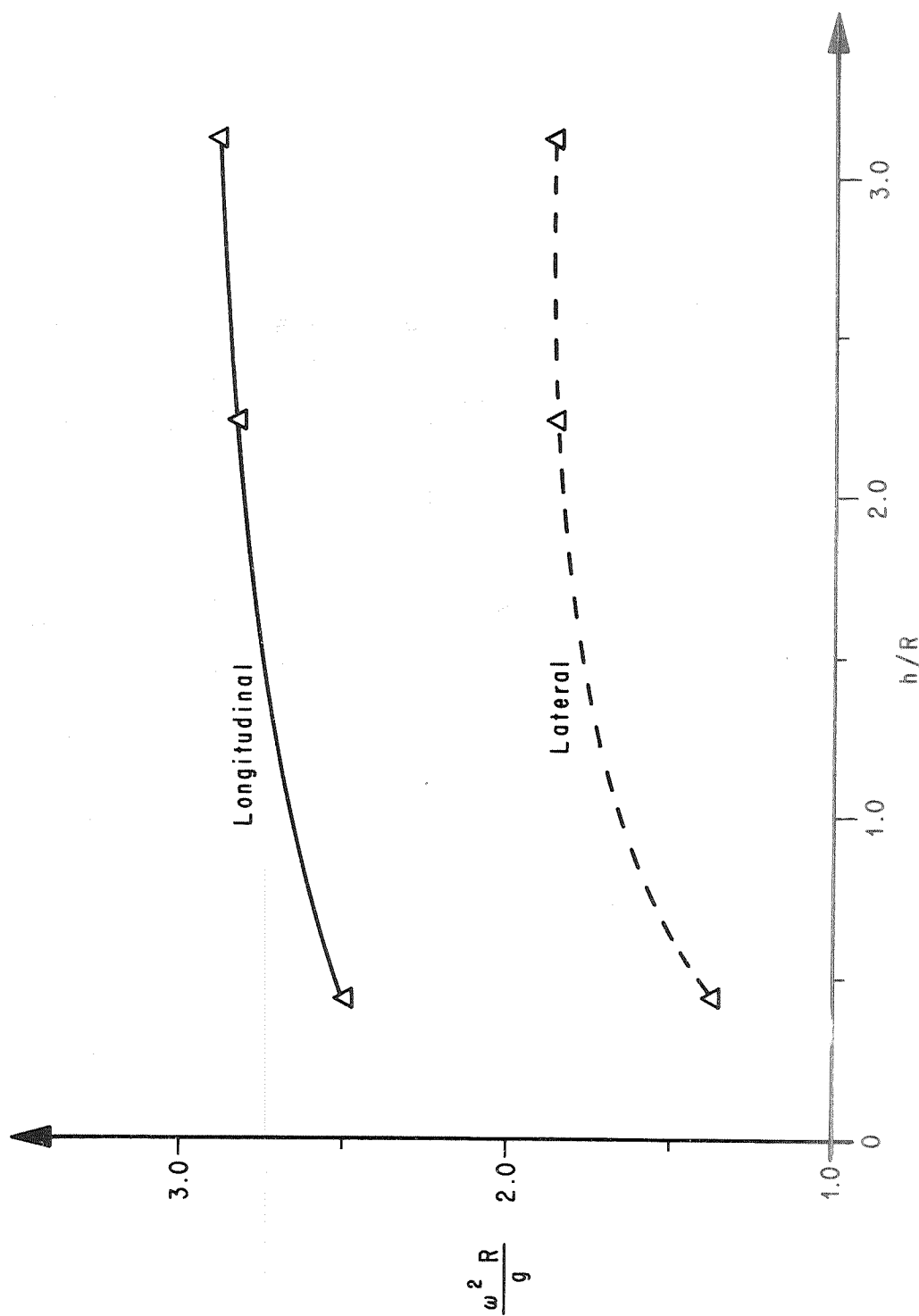


FIG. 9. EFFECT OF LIQUID DEPTH  
SHAPE 4, ORIENTATION 2,  $\alpha = 10^\circ$

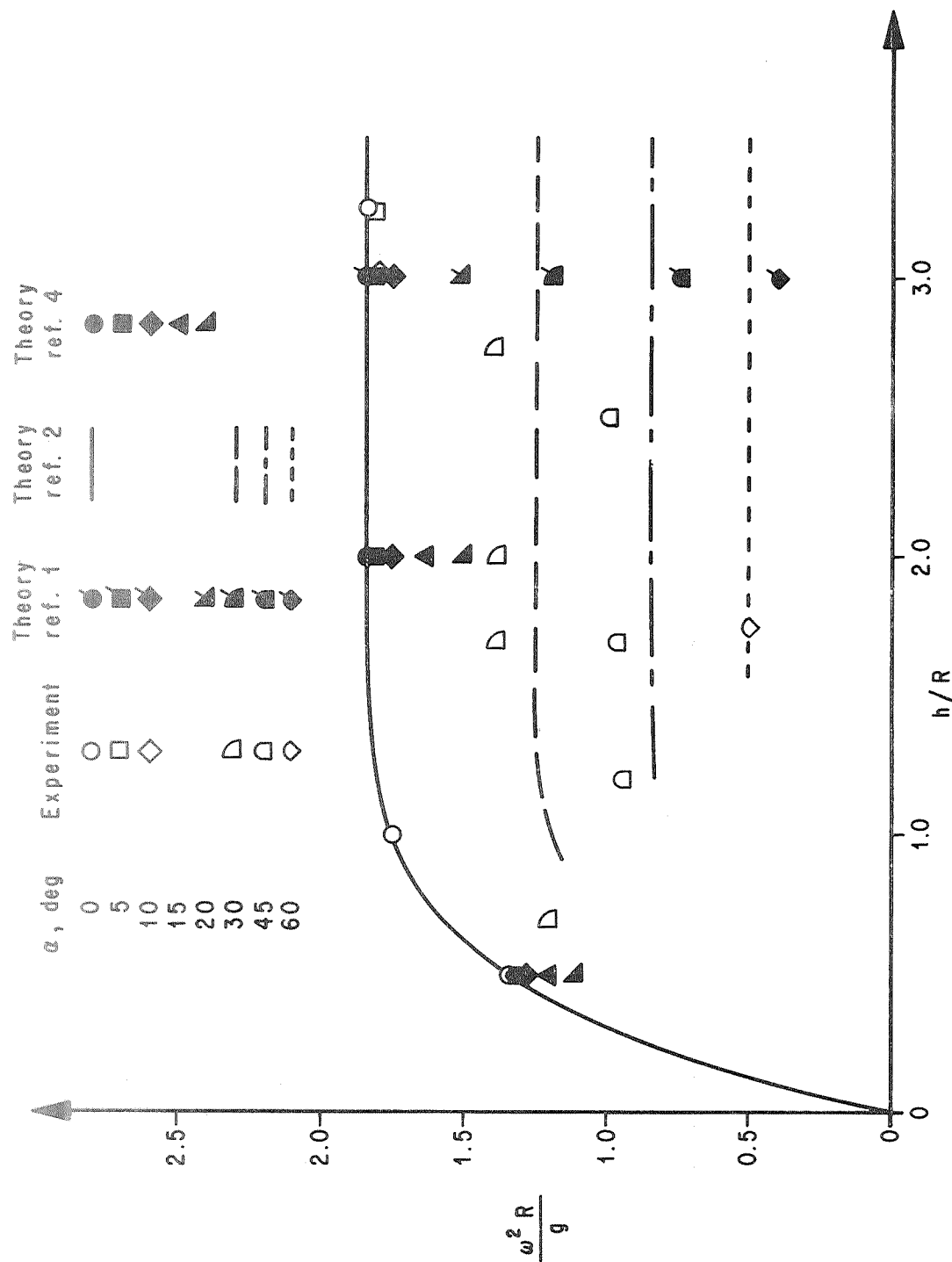


FIG. 10. COMPARISON OF EXPERIMENT AND THEORY, LONGITUDINAL MODE, CIRCULAR CROSS SECTION


APPROVAL

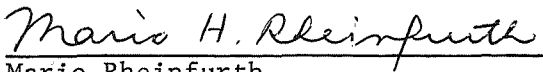
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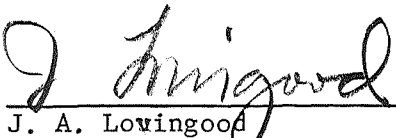
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